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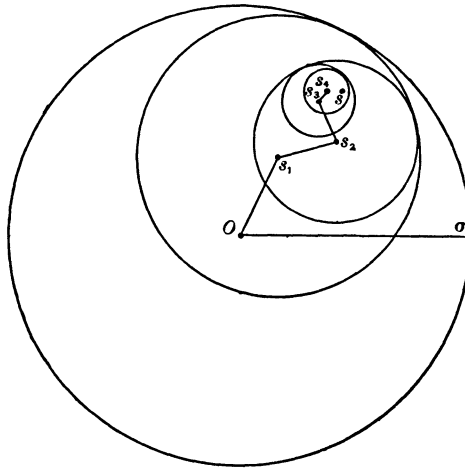
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A GEOMETRIC DISCUSSION OF THE ABSOLUTE CONVERGENCE OF A SERIES WITH COMPLEX TERMS

BY GEORGE H. LING

THAT a clear understanding of a great many points in connection with questions of convergence of series is facilitated by the use of geometric figures would seem to be the opinion of all who have used such diagrams either in private study or in exposition of the subject in the class-room. The following use of a diagram will, it is thought, prove valuable in this connection.

The theorem to be illustrated is a very important one.



THEOREM. *The infinite series of complex terms $\sum_{r=1}^{\infty} u_r$ is convergent if the infinite series $\sum_{r=1}^{\infty} |u_r|$ is convergent.*

The formal analytic proof is accessible to all and is omitted. The geometric illustration is as follows, the ordinary geometric representation of complex quantities being used.

Let

$$\sum_{r=1}^n |u_r| = \sigma_n, \quad \sum_{r=1}^{\infty} |u_r| = \sigma, \quad \sum_{r=1}^n u_r = s_n.$$

In the plane of the complex variable describe a circle Γ whose centre is the origin and whose radius is σ . (See figure.)

The point s_1 lies within or on the circumference Γ . Now with centre s_1 and radius equal to $\sigma - |u_1| = \sigma - \sigma_1$ describe a circle C_1 . The point s_2 lies within or on the circumference C_1 . With centre s_2 and radius equal to $\sigma - |u_1| - |u_2| = \sigma - \sigma_2$ describe a circle C_2 . The point s_3 lies within or on the circumference C_2 . The construction of circles may be continued as far as desired. About the point s_n as centre there will be described a circle C_n having the radius $\sigma - \sigma_n$.

In this construction, each circle is tangent internally to the one described just before it. Any circle C_n contains the point s_n and all succeeding points of the set

$$(1) \qquad s_1, s_2, s_3, \dots, s_n, \dots;$$

and by taking n sufficiently large the radius of the circle C_n may be made smaller than any assigned positive quantity.

Therefore the points of the set (1) approach a limiting point s , within all of the circles, the remaining steps of the argument being readily supplied.

In conclusion, it should be noticed, that the argument cannot be applied to the case of a conditionally convergent series because in such a case the circles $\Gamma, C_1, C_2, \dots$ are all of infinite radius.

COLUMBIA UNIVERSITY,
NEW YORK.